

# Gödel's Modal Ontological Proof

Jason Zhang

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## Abstract

We go over an English version of Kurt Gödel's Modal Ontological Proof for the existence of God. Such an argument is a strictly mathematical one, and it is true by the sense of formal logic *a priori* rather than physical evidence. The purpose of this document is to recount this argument by Gödel. It should be noted that modern philosophers have both criticisms and strengthenings of this argument that may either weaken or improve its credibility. Regardless, we will not cover any modern developments.

## 1 Modal Logic

The system the proof takes place in is *modal logic*, which is slightly different than classical logic used by classical mathematics. Namely, there are two additional symbols (which can be thought of as quantifiers) which are  $\Box$  and  $\Diamond$ , respectively interpreted as “necessarily” and “possibly”. The original proof involved pure symbolic manipulations, so one must be familiar with these symbols. However, since we are presenting in English, we will say “necessarily” and “possibly” unless the symbols are needed.

In our semantic framework, these two operators refer to possible worlds. We now can translate the  $\Box, \Diamond$  to  $\forall, \exists$ . Simply, “necessarily” is the same as “in all possible worlds” and “possibly” is the same as “there exists a possible world” (in fact,  $\Box, \Diamond$  heavily behave like  $\forall, \exists$ ). Now, unless one admits modal realism, these possible worlds usually do not exist (so there is **no** multiverse).

**Example 1.** It is possible that you have a cat as a pet.

Regardless of whether you actually have a pet cat or not, it should be believable that this is possible (ignoring extreme cases or philosophies). This is a use case of the possibility operator.

**Example 2.** It is necessary that ice melts into water.

Indeed, in any possibly situation, ice would melt into water (again ignoring malicious counterexamples; this example is to illustrate a rough idea). This is a use case of the necessary modal operator.

*Remark.* It is notable that this ontological proof may be best studied from that of a second order perspective. Indeed, we will be presenting properties of properties. Also, we will be working in S5 but this is not too important to study in depth.

The nature of this proof is to attempt to define some universally agreed upon definitions and axioms, and then conclude forcibly that God exists. It is important to note that this proof does not decide which religion is correct, just that God must exist. Furthermore, this version is technically due to Dana Scott, who added a correction upon the original.

## 2 The Proof

We first define a predicate that acts as “positivity”. The semantics of which are not particularly important to this proof, but they are important to whether you accept or reject the axioms this proof is founded upon.

**Definition 2.1.** We call the (second order) predicate  $P$  to be “positive”. For some other predicate  $\varphi$ ,  $P(\varphi)$  reads “ $\varphi$  is a positive property”.

**Axiom 2.2.** Let  $\varphi$  be a positive property. If it is necessary that every object  $x$  with property  $\varphi$  also has property  $\psi$ , then  $\psi$  is also positive.

*Remark.* What this means is that if some property  $\varphi$  is positive, and it always leads to another property  $\psi$ , then  $\psi$  must also be positive. In other words, “a positive property cannot always lead to a negative property”.

**Axiom 2.3.** For any  $\varphi$ , the negation  $\neg\varphi$  is positive if and only if  $\varphi$  is not positive.

**Lemma 2.4.** *If  $\varphi$  is a positive property, then it is possible that there exists some  $x$  such that  $x$  has property  $\varphi$ .*

*Proof.* Assume for the sake of contradiction that  $\varphi$  was positive or good, but it was not possible that there exists some  $x$  with property  $\varphi$ . Symbolically, this would look like  $\neg\Diamond\exists x \varphi(x)$ . Since it is not possible that there exists an  $x$ , we can conclude (both by thinking about it and by modal inference rules) that it is necessary that every  $x$  does not have property  $\varphi$ . That is,  $\Box\forall x \neg\varphi(x)$ . Since  $\varphi$  holds nowhere, it vacuously implies any property  $\psi$  to be true. By Axiom 2.2, this means that  $\varphi$  forces ANY property  $\psi$  to be a positive property, including  $\neg\varphi$ . By Axiom 2.3, if  $\neg\varphi$  is positive this means  $\varphi$  is not positive, contradiction. Therefore, if  $\varphi$  then it is possible that some  $x$  could instantiate it.  $\square$

**Definition 2.5.** We say that  $x$  is God (or God-like) if and only if  $x$  has every positive property. This itself is a property/predicate notated as  $G(x)$ .

**Axiom 2.6.** Being God is good. In other words,  $G$  is a positive property.

**Lemma 2.7.** *It is possible that there exists  $x$  that is God-like.*

*Proof.* By Lemma 2.4, every positive property possibly has some instantiation. Directly applying it, we find that the property  $G$  must follow this as well. It follows that it is possible that there exists  $x$  such that  $G(x)$  or “ $x$  is God”.  $\square$

**Definition 2.8.** We call property  $\varphi$  the “essence” (or an essential property) of  $x$  whenever

- (i)  $x$  has property  $\varphi$ ;
- (ii) If  $x$  has some other property  $\psi$ , then necessarily  $\psi$  follows from  $\varphi$  for all objects.

**Axiom 2.9.** If  $\varphi$  is positive, then it is necessarily positive in all worlds.

**Lemma 2.10.** *If  $x$  is God, then being God-like is an essential property to  $x$ .*

*Proof.* By definition, God has every positive or good property and none of the bad ones. Any property  $\varphi$  God has is therefore good, and also necessarily good by Axiom 2.9, which implies it is necessarily possessed by God.  $\square$

**Definition 2.11.** An object  $x$  is “indispensable” if and only if each of its essential properties applies in every possible world to some object  $y$ .

**Axiom 2.12.** Being indispensable is a positive property.

**Theorem 2.13.** *God exists, necessarily.*

*Proof.* If  $x$  is God, then  $x$  has every good property by definition. It is therefore indispensable by Axiom 2.12. That property implies that every essential property must be instantiated in every possible world. But, by Lemma 2.10, one of  $x$ ’s essential properties was being God. So, if there exists  $x$  with the God property, then necessarily there exists an  $x$  with the God property. By Lemma 2.7, we know it is possible that God exists. It follows that it is possible that God exists necessarily. Since every possible world ranges over the same collection of possible worlds, if there is one possible world from which God exists in every possible world, then God indeed exists in every possible world. Therefore, God exists in every possible world, including the actual world. This completes the proof.  $\square$